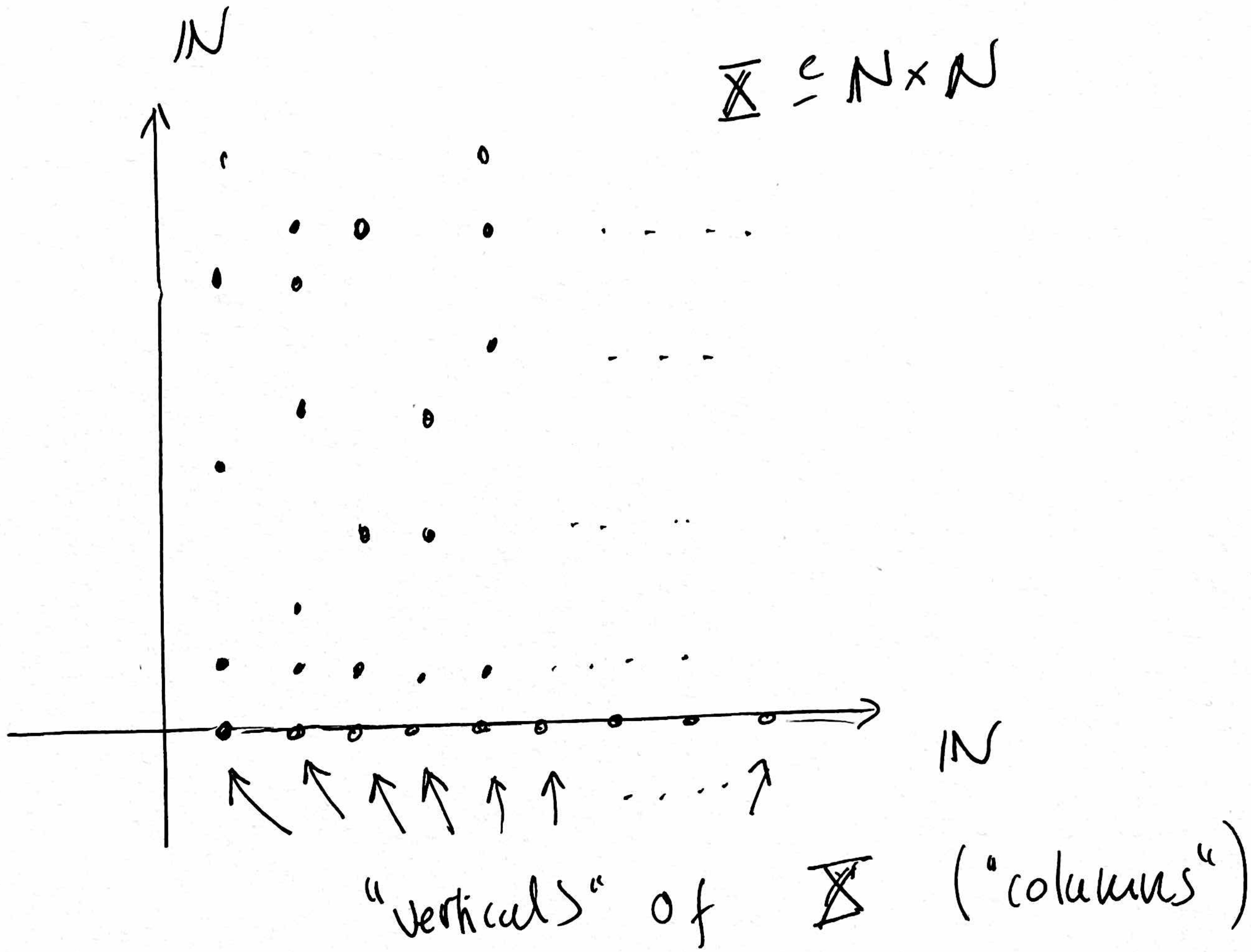
P.1

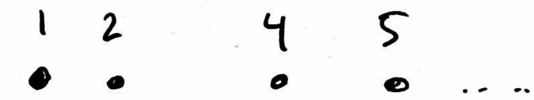
To slide "Notation"



$X \in FIN^2$ means: X has only finitely many infinite verticals.

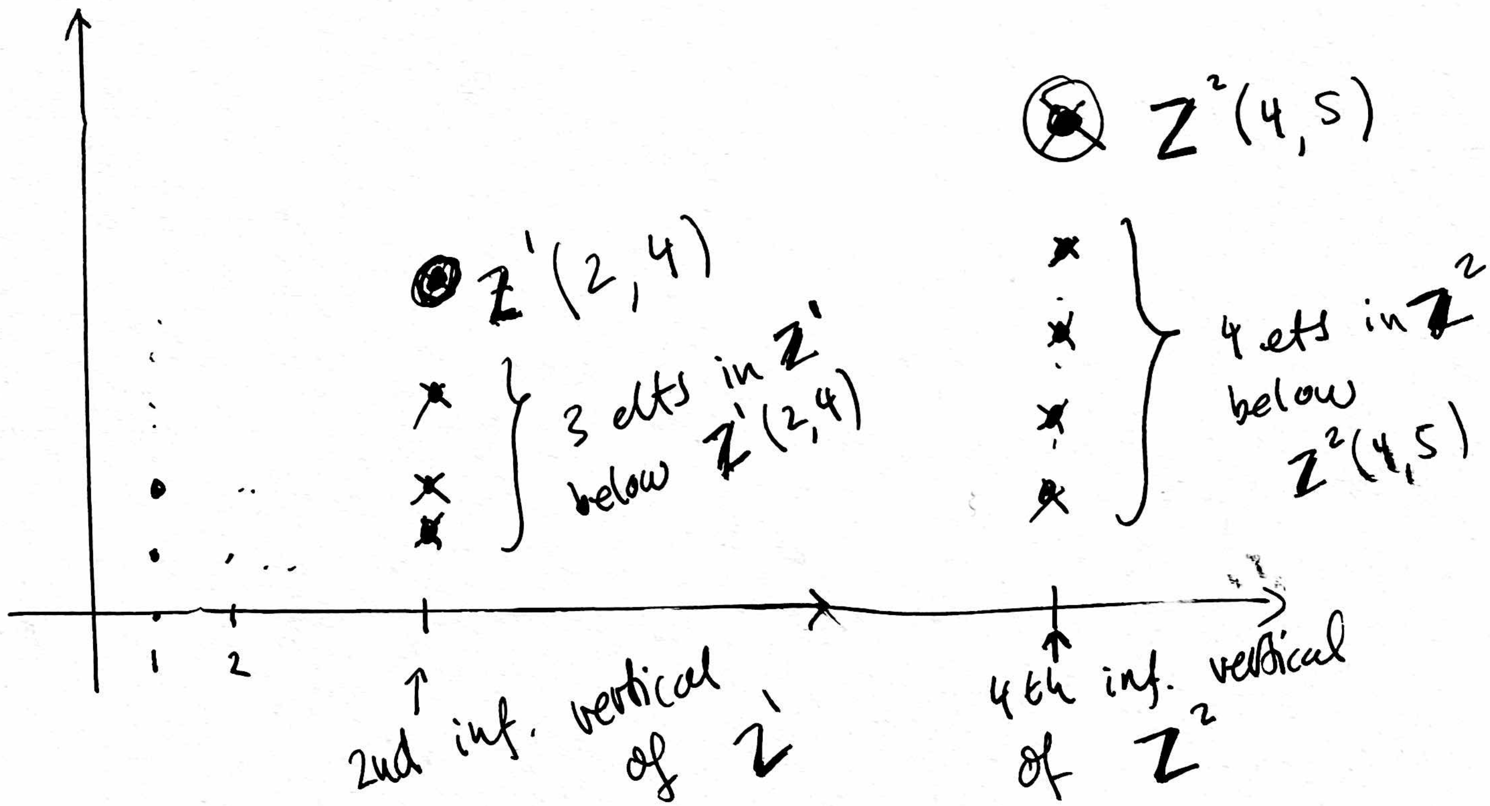
$\# FIN^{2+} = \# \mathcal{P}(N \times N) \setminus FIN^2$ = the co-ideal p.2 To slide "from 1 to 2 dim"

Example:



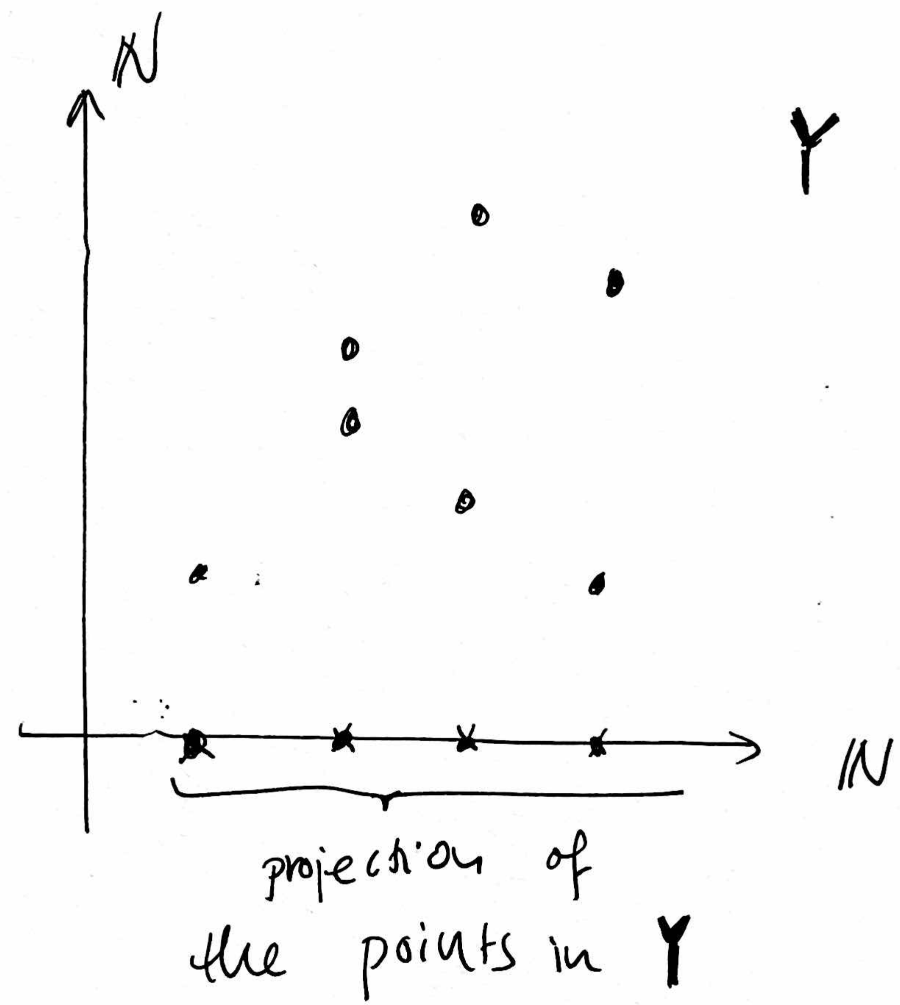
- $A \subseteq \mathbb{N}$ given (and infinite):
- Z^ℓ, $\ell \in \mathbb{N}$ our fixed sequence of elements of the family $\mathcal{U}$.
- $\hat{Z}^\ell(m,n)$ is the n'th element of the m'th vertical (column) of Z^ℓ

Example:



To slide "Facts about the tilde op."

For $Y \subseteq \mathbb{N}^2$, let $\mathrm{dom}(Y) \subseteq \mathbb{N}$ be the projection of Y onto the first coordinate:



To slide: "The associated trees T^X and $T^{X,d}$".

p.4

Idea: We can think of $p[T_t]$ as a subfamily of $\mathcal{A} = p[T]$ which comes from the "localized" part of the tree (at t), i.e. T_t.

- $T^{X} \subseteq T$ is the subtree of those $t \in T$ where there is some $A \in p[T_t]$ s.t.

$$A \cap X \in FIN^{2+}$$

- $T^{X,d}$ even requires that the verticals $(A \cap X)_i$ are infinite for $i \in d$.

(to slide: ~~...~~ "Putting it all together: Claim 2 cont'd")

p.5 Let $\mathrm{dom}_\infty(X) = \{ i \in \mathbb{N} \mid X(i) \text{ is infinite} \}$.

- If ① happens, then $|\mathrm{dom}_\infty(X_0) \cap \mathrm{dom}_\infty(X_1)| < \infty$ and so by pigeon hole principle ① we can find $B \in [d, A]$ s.t.

(*) $$\mathrm{dom}_\infty(\tilde{B}) \cap \mathrm{dom}_\infty(X_0) \text{ is finite.}$$

  This is impossible since $T^{\tilde{B},d} = T^{\tilde{A},d}$ and so $X_0 \in p[T^{\tilde{B},d}_{t_0}] = p[T^{\tilde{A},d}_{t_0}]$, contradicting the definition of $T^{\tilde{B},d}_{t_0}$

- If ② happens then pigeon hole principle ② gives us $B \in [d, A]$ s.t. for some $i \in d$,
$$\tilde{B}(i) \cap X_0(i) \text{ is finite}$$
  This is also impossible since $T^{\tilde{B},d} = T^{\tilde{A},d}$ and $X_0 \in p[T^{\tilde{A},d}_{t_0}]$