On the universal minimal flow of the homeomorphism group of a Knaster continuum

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Outline

Outline

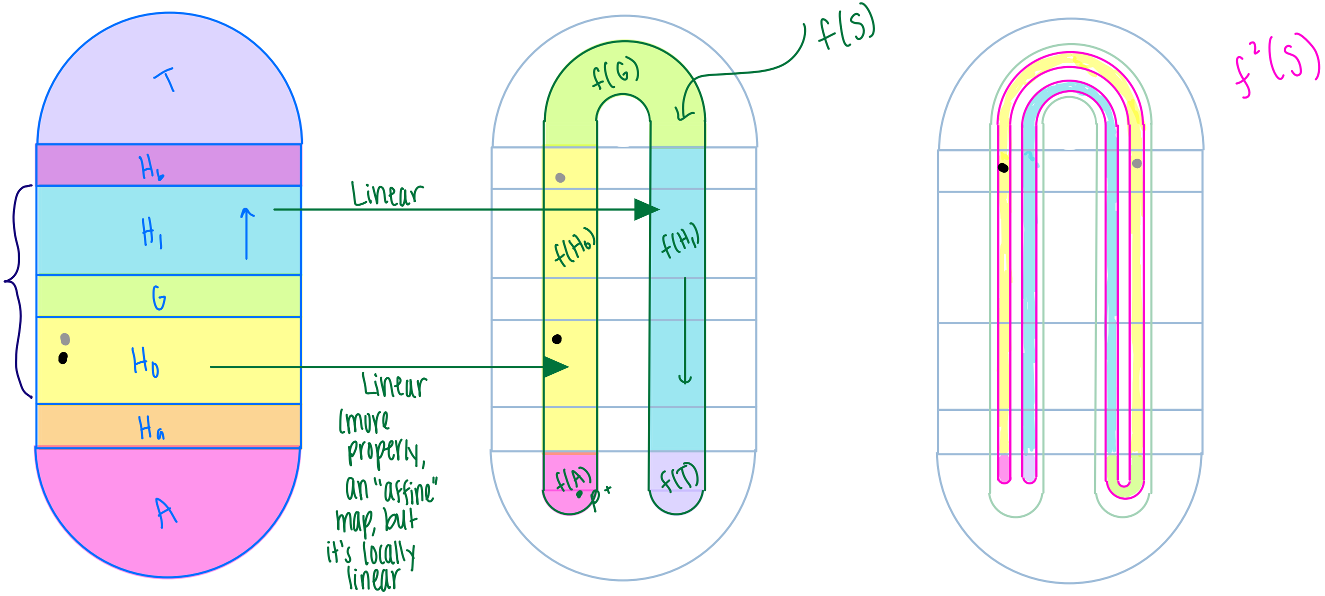
Smale & The Horseshoe Map

- Stephen Smale, 1960 in Rio: So maybe chaos DOES exist
  - Mary Cartwright & L. J. Littlewood worked on analyzing equations that arose in the study of radio waves in World War II
  - Norman Levinson (MIT) found in their work an example of a chaotic system
  - Smale then worked to represent this example in as simple a manner as possible, and came up with the Horseshoe map (Won Fields Medal 1966)
  - The **attractor** of the Smale horseshoe map is an **indecomposable continuum**.

$$K_2 = \bigcap_{n=0}^{\infty} f^n(D^2)$$

"A system is chaotic if the future is determined by the present, but the approximate present does not approximate the future."

The Horseshoe Map



Indecomposable Continua

Indecomposable Continuum

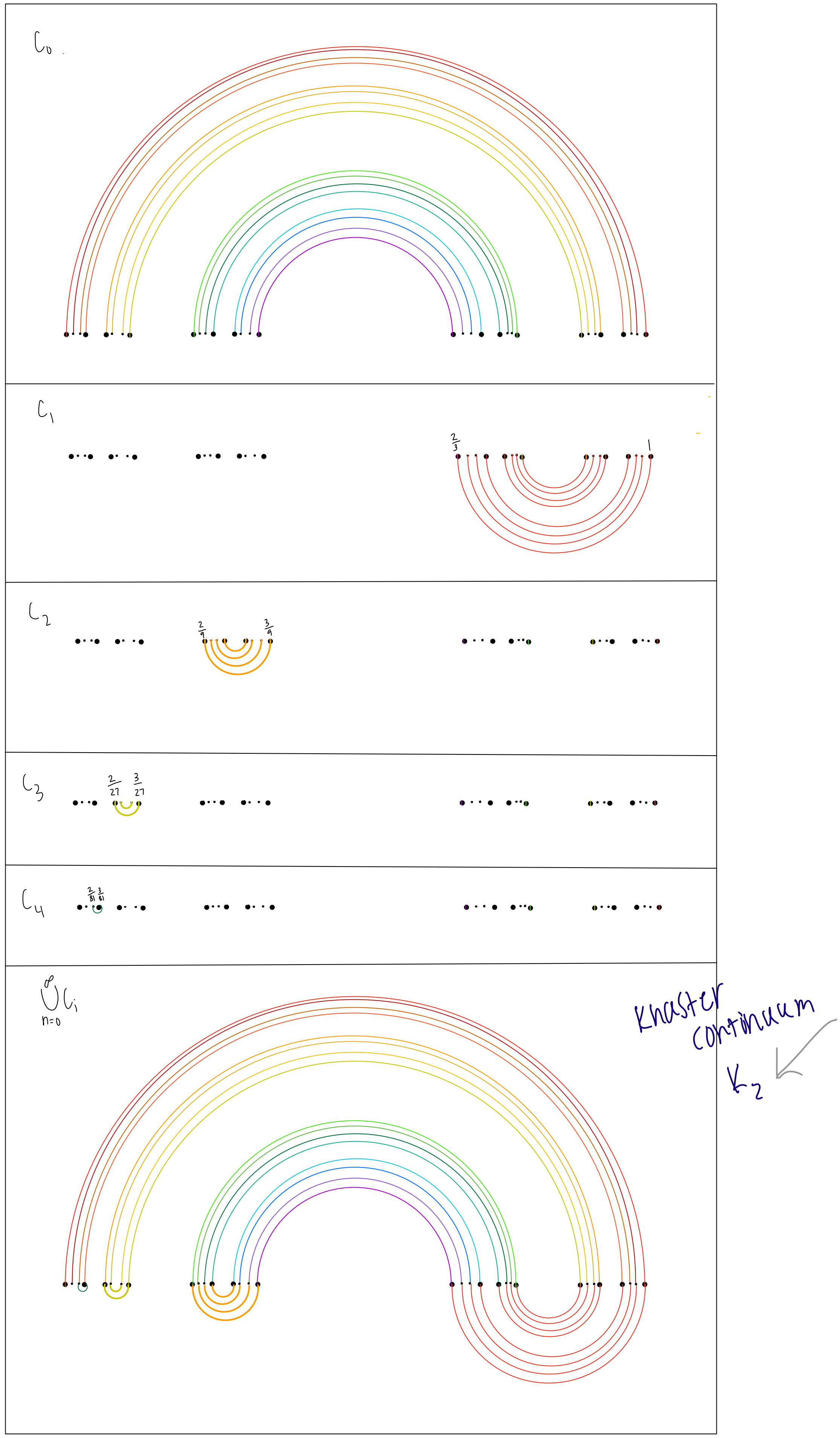
Say a continuum is decomposable if it can be written as a union of two proper subcontinua. Otherwise, say the continuum is indecomposable.

- In 1910, L. E. J. Brouwer was the first to describe an indecomposable continuum.
- The Buckethandle example first appeared in Kuratowski's 1922 continuum theory paper.
- In the paper he attributed the construction to Knaster, who in turn obtained his definition by simplifying Janiszewski's definition from his thesis

The Geometric Description

This description comes from Kuratowski's 1968 textbook *Topology II*:

1. Form a set $\mathcal{C}_0$ which is defined to be the set of all semi-circles in ^{2} that lie above the x-axis, whose endpoints are elements of the Cantor (middle-third) set, and whose center is $(\frac{1}{2}, 0)$
2. Next, form the set $\mathcal{C}_1$ which is defined to be the set of all semi-circles in ^{2} that lie **below** the x-axis, whose endpoints are elements of the Cantor set, and whose center is the midpoint of the interval $[\frac{2}{3}, \frac{3}{3}]$.
3. Continue so that each $\mathcal{C}_n$ is the set of semi-circles lying **below** the x-axis, whose endpoints are Cantor points, and whose center is the midpoint of the interval $[\frac{2}{3^n}, \frac{3}{3^n}]$. Finally, form the Knaster continuum by taking the union of all of these sets, $K = \bigcup_{n=0}^{\infty} \mathcal{C}_n$.



C_0
C_1
$\frac{2}{3}$
1
C_2
$\frac{2}{9}$
$\frac{3}{9}$
C_3
$\frac{2}{27}$
$\frac{3}{27}$
C_4
$\frac{2}{81}$ $\frac{3}{81}$
$\bigcup_{n=0}^{\infty} C_i$
Khaster continuum
K_2

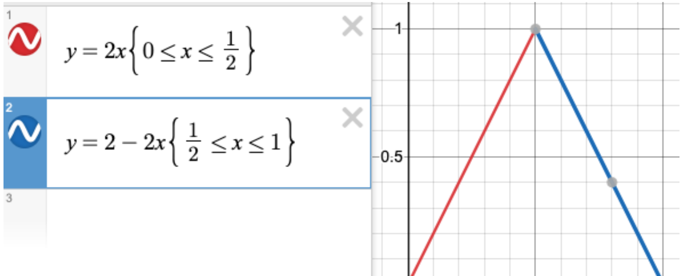
The Brouwer-Janiszewski-Knaster continuum

The Knaster Continuum

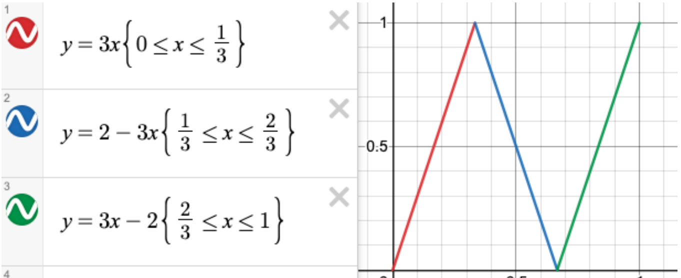
The Knaster Continuum first presented in 1922 (which we will denote K_2 from now on) is homeomorphic to the inverse limit space of a certain tent map on the unit interval. We will call this particular tent map $f_2 : I \to I$, and it is defined as follows (See example 22, pg 15 of Ingram and Mahavier's *Inverse Limits*):

$$f_2(x) = \begin{cases} 2x \text{ if } x \in [0, \frac{1}{2}] \\ 2(1-x) \text{ if } x \in [\frac{1}{2}, 1] \end{cases}$$

2-Tent Map



3-Tent Map



The Brouwer-Janiszewski-Knaster continuum

The Knaster Continuum

There exists a whole class of mutually non-homeomorphic Knaster continua. Each are homeomorphic to an inverse limit space of a sequence of tent maps on the unit interval.

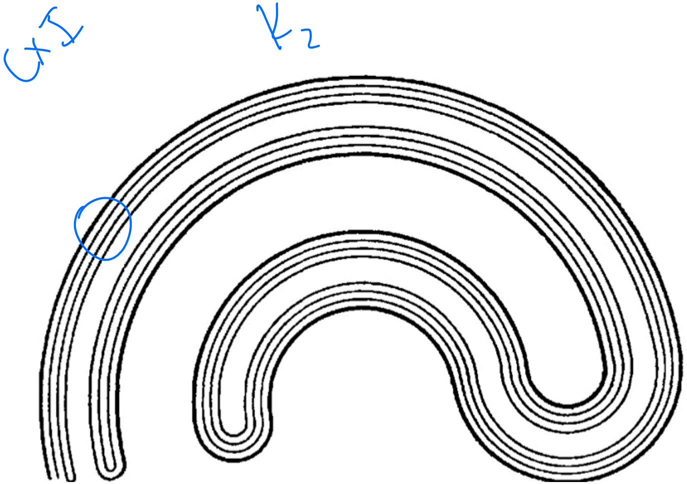
$$K_2 = \varprojlim(I, f_2) = \{(\ldots, x_2, x_1, x_0) \in I^{\cdot} x_i = f_2(x_{i-1})\}$$

$$\begin{tikzcd}
 & & K_2 \arrow[dll] \arrow[dl] \arrow[d] \arrow[dr] \arrow[drr] & & \\
I & I \arrow[l, "f_2"'] & I \arrow[l, "f_2"'] & I \arrow[l, "f_2"'] & \cdots \arrow[l, "f_2"']
\end{tikzcd}$$

f_3 f_5 f_2

$f_2 \circ f_3 = f_6$

K_2 and K_3



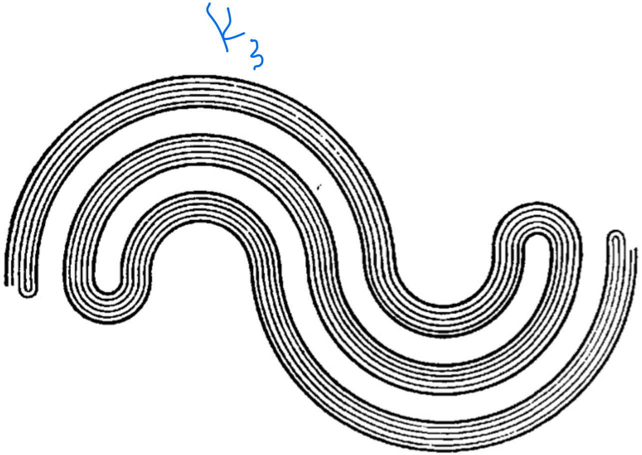


Image from K. Kuratowski's 1968 Textbook *Topology II*, page 205

The Lattice

$K_{2,3,5} \longrightarrow K_{2,3}$

Debski's Classification, 1985

Two Knaster Continua K_π and K_ρ are homeomorphic if and only if for all but finitely many primes p, p occurs in the sequences π and ρ the same number of times. In the exceptional cases, the number of occurences of p in each sequence is finite.

$K_\alpha < K_\beta \Leftrightarrow \exists \varphi : K_\beta \to K_\alpha$

Theorem, Eberhart, Fugate, & Schumann, 2002

There exists a lattice ordering on the equivalence class of Knaster continua with K_τ at the top (where τ is the sequence with infinitely many occurrences of every prime), K_1 at the bottom, where (1) is any sequence of primes with all but finitely many 1's.

K_ρ

K_1

Let $\rho = (2, 3, 2, 3, 5, 2, 3, 5, 7, \ldots)$ be the sequence of primes where every prime occurs infinitely many times. Then K_ρ is universal in the class of Knaster continua, in that it maps openly onto every other Knaster continuum.

Abstract Characterization of a Knaster Continuum

Theorem, Krupski 1984

The class of all Knaster continua is equal to the class of all arc-like continua with one or two endpoints having the **Property of Kelley** and arcs as non-degenerate subcontinua and which themselves are not arcs. In particular, all of them are indecomposable.

Property of Kelley

A continuum X has the Property of Kelley if for all $x \in X$ and for each subcontinuum K of X containing x and for each sequence of points (x_n) with $x_n \to x$, there exists a sequence of subcontinua K_n of X containing x_n and converging to K.

Kelley conntinuum

Homeo(K)

There is much known about the homeomorphism group of the Knaster continuum (K_2), in particular, the types of homeomorphisms that exist, and the number of fixed points they must have, and the fact that the homeomorphism group is indeed a topological group (Ssembatya, 2001 under Keesling). However, several questions remained, many of which can be answered using projective Fraïssé theory:

- What kind of subgroups does the homeomorphism group have?
- Is the group locally compact? NO
- Can the homeomorphism group be generated by neighborhoods of the identity?
- What is the universal minimal flow?

Outline

Epimorphisms Instead of Embeddings

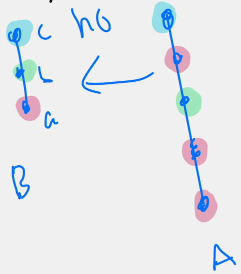
Irwin & Solecki, 2006

Let $\mathcal{L}$ be a language consisting of relation symbols R_i, $i \in I$, with arity $m_i \in$, and function symbols $f_i, j \in J$ with arity $n_j \in$. A topological $\mathcal{L}$-structure is a zero-dimensional, compact, second countable space A together with closed sets $R_i^A \subseteq A^{m_i}$ and continuous functions $f_j^A : A^{n_j} \to A$ for all $i \in I$ and $j \in J$.

Irwin & Solecki, 2006

An epimorphism from A to B is a surjective, continuous function $\varphi : A \to B$ such that for any $j \in J$ and $x_1, \ldots, x_{n_j} \in A$ we have:

$$f_j^B(\varphi(x_1), \ldots, \varphi(x_{n_j})) = \varphi(f_j^A(x_1, \ldots, x_{n_j}))$$



and for any $i \in I$ and any $y_1, \ldots, y_{m_i} \in B$ we have:

$$(y_1, \ldots, y_{m_i}) \in R_i^B \Leftrightarrow \exists x_1, \ldots, x_{m_i} \in A\,(\varphi(x_p) = y_p \text{ for all } p \leq m_i \text{ and } (x_1, \ldots, x_{m_i}) \in R_i^A$$

Projective Fraïssé Classes

$$\begin{tikzcd} & F \arrow[dl, dashed] \arrow[dr, dashed] & \\ D & & E \end{tikzcd}$$

2006

Projective Fraïssé Class, Irwin & Solecki

Let $\mathcal{D}$ be a family of topological $\mathcal{L}$-structures. We say that $\mathcal{D}$ is a projective Fraïssé class if the following two conditions hold:

(F1) For any $D, E \in \mathcal{D}$, there is an $F \in \mathcal{D}$ and epimorphisms from F onto D and onto E.

(F2) For any $C, D, E \in \mathcal{D}$ and any epimorphisms $\varphi_1 : D \to C$, $\varphi : E \to C$, there exists an $F \in \mathcal{D}$ with epimorphisms $\psi_1 : F \to D$ and $\psi_2 : F \to E$ such that $\varphi_1 \circ \psi_1 = \varphi_2 \circ \psi_2$

$$\begin{tikzcd} & F \arrow[dl, dashed, "\psi_1"'] \arrow[dr, dashed, "\psi_2"] & \\ D \arrow[dr, "\varphi_1"'] & & E \arrow[dl, "\varphi_2"] \\ & C & \end{tikzcd}$$

Projective Fraïssé Class

Projective Fraïssé Class, Charatonik & Roe, 2020

Let $\mathcal{F}$ be a class of finite graphs with a fixed family of morphisms among the structures in $\mathcal{F}$. We assume that each morphism is an epimorphism with respect to $\mathcal{F}$. We say that $\mathcal{F}$ is a projective Fraïssé class if:

1. $\mathcal{F}$ is countable up to isomoprhism, that is, any sub-collection of pairwise non-isomoprhic structures of $\mathcal{F}$ is countable;
2. morphisms are closed under composition and each identity map is a morphism;
3. for $B, C \in \mathcal{F}$ there exists $D \in \mathcal{F}$ and morphisms $f : D \to B$ and $g : D \to C$; and
4. for every two morphisms $f : B \to A$ and $g : C \to A$, there exist morphisms $f_0 : D \to B$ and $g_0 : D \to C$ such that $f \circ f_0 = g \circ g_0$.

$\mathcal{D} \mapsto \mathbb{D}$

2006

Projective Fraïssé Limit, Irwin & Solecki

Let $\mathcal{D}$ be a family of topological $\mathcal{L}$-structures. We say that a topological $\mathcal{L}$-structure $\mathbb{D}$ is a projective Fraïssé limit of D if the following three conditions hold:

(L1) (projective universality) for any $D \in \mathcal{D}$, there is an epimorphism from $\mathbb{D}$ to D.

(L2) for any finite discrete topological space A and any continuous function $f : \mathbb{D} \to A$, there is a $D \in \mathcal{D}$ an epimorphism $\varphi : \mathbb{D} \to D$, and a function $\overline{f} : D \to A$ such that $f = \overline{f} \circ \varphi$.

(L3) projective ultrahomogeneity for any $D \in \mathcal{D}$ and any epimorphisms $\varphi_1 : \mathbb{D} \to D$ and $\varphi_2 : \mathbb{D} \to D$, there exists an isomorphism $\varphi : \mathbb{D} \to \mathbb{D}$ such that $\varphi_2 = \varphi_1 \circ \psi$.

$\mathcal{F} : \overset{\mathbb{D}}{\Delta} \to \overset{\mathbb{D}}{\Delta}_0$

$\mathcal{G} \subseteq \Delta$ $\quad \varphi$

Charatonik & Roe 2020

The Family of Finite Linear Graphs

Let $\mathcal{L} = R$, the language consisting of one binary relation R. Let Δ be the class of all finite (reflextive) linear graphs, i.e. the class of all finite sets A with at least two elements so that R^A has the following properties:

- R^A is reflexive;
- R^A is symmetric;
- Every element of A has at most three (including itself) R^A-neighbors;
- There are exactly two elements of A with less than three R^A neighbors;
- R^A is connected, i.e. for every $a, b \in A$ there exists $a_0, \ldots, a_n \in A$ such that $a = a_0, b = a_n$ and $(a_i, a_{i+1}) \in R^A$ for $0 \leq i < n$.

Theorem (Irwin & Solecki, 2006)

The Family of Finite Linear Graphs forms a Projective Fraïssé Class.

$\Delta/\sim \simeq \mathbb{P}$

Compact Spaces & Quotients of PF Limits

Every compact space is homeomorphic to a quotient of a projective Fraïssé limit

$Y = \mathbb{K}/R^{\mathbb{K}}$

Theorem (Panagiotopoulos, 2017)

Let G be a closed subgroup of Homeo(Y) for some compact, metrizable space Y. Then there is a projective Fraïssé pre-space $\mathbb{K}$ such that $\mathbb{K}/R^{\mathbb{K}}$ is homeomorphic to Y, and the quotient projection

$$\mathbb{K} \mapsto Y$$

induces a continuous group embedding $\text{Aut}(\mathbb{K}) \hookrightarrow G$ with dense image in G.

$\mathcal{L} = \bigcup \mathcal{L}^n$

$\text{Homeo}(K) \supseteq [n]^K$

$\mathcal{L}^n \; \{ O_i^n : i \in I^n \}$

Easier Any compact space $K = G/R$ This is a topological graph.

Pangio showed Projective Fraïssé space

$f : \{0,1\}^{\mathbb{N}} \to K$

$(\vec{a}, \vec{b}) \in R$

$G = \{\{0,1\}^{\mathbb{N}}, R\}$ $\iff f(\vec{a}) = f(\vec{b})$

$G_n =$

All Possible Quotients

Below is a characterization of all possible quotients from a language with one binary relation: Finitely representable compact space

Theorem (Camerlo, 2010)

Given a language $\mathcal{L} = R$ which consists of a single binary relation. The characterization of all spaces that can be obtained, up to homeomorphism, as quotients $\mathbb{P}/R^{\mathbb{P}}$, where $(\mathbb{P}, R^{\mathbb{P}})$ is the projective Fraïssé limit of a projective Fraïssé family of finite topological $\mathcal{L}$-structures, for $\mathcal{L}$ as above and $R^{\mathbb{P}}$ an equivalence relation consists of the following list:

- Cantor space;
- Disjoint unions of m singletons and n pseudo-arcs, with $m + n > 0$
- Disjoint sums of n spaces each of the form $X = P \cup \bigcup_{j\in\mathbb{N}} Q_j$ where P is a pseudo-arc, and Q_j is a Cantor space which is clopen in X and $\bigcup_{j\in} Q_j$ is dense in X.

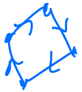
"Topological realization"

Class	Epimorphisms	Quotient of Limit	Attribution
Linear Graphs	All	Pseudo-arc	Irwin & Solecki (2006)
Linear (Singly Directed) Graphs	Order-Preserving	Arc	Basso & Camerlo (2016)
Finite Graphs	Monotone	Menger Curve	Pangiatopoulos & Solecki (2020)
Connected Graphs	All	Does not amalgamate	W. J. Charatonik & Roe (2020)
Connected Graphs	Confluent	Unnamed	W. J. Charatonik & Roe (2020)
Trees	All	Does not amalgamate	W. J. Charatonik & Roe (2020)
Trees	Monotone	D_3	W. J. Charatonik & Roe (2020)
Trees	Monotone Retractions	A special dendrite called P – No quotient	W. J. Charatonik & Roe (2020)
Trees	Confluent	Does not amalgamate	W. J. Charatonik & Roe (2020)
Rooted Trees	Order-Preserving	Lelak Fan	Bartošová & Kwiatkowsa (2013)
Rooted Trees	Order-Preserving & End-Preserving	Cantor Fan	W. J. Charatonik & Roe (2020)
Rooted Trees	Confluent & Order-Preserving	Unnamed	W. J. Charatonik & Roe (2020)
Rooted Trees	Confluent & Order-Preserving & End-Preserving	Mohler-Nikiel Dendroid	W. J. Charatonik & Roe (2020)
Hasse Diagrams of Finite Partial Orders	Various*	Fences	Camerlo & Basso (2020)
Directed Linear Graphs	Open Wrapping Epimorphisms	Universal Knaster Continuum	W. & Bartošová (2021)

$L = \{R, \leq\}$

Fence = compact metrizable spaces whose connected components are arcs & points

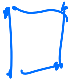
Future Results



$\mathbb{I}$

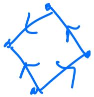
- Circle S^1
  - Objects: Singly Directed Cycles (every element has exactly one successor and exactly one predecessor)
  - Epimorphisms: Order-Preserving

pseudo-



$\mathbb{P}$

- Universal Solenoid Σ
  - Objects: Directed Cycles (every element has degree 3 including itself)
  - Epimorphisms: All



K_ρ

- Universal Solenoid Σ_ρ
  - Objects: Directed Cycles (every element has degree 3 including itself)
  - Epimorphisms: Order-Preserving, Open, Confluent

Say $f: G \to H$ is confluent if every connected subset $Q \subseteq V(H)$ and for every component $C \subseteq f^{-1}(Q)$, have $f(Q) = C$

Outline

Describing the Approximating Class

The class of Finite Linear Singly Directed Graphs with Open Wrapping Epimorphisms

confluent & open

Theorem, W. & Bartošová, 2021

The following class together with open wrapping epimorphisms forms a projective Fraïssé class: Let $\mathcal{L} = \{\leq\}$ be a first order language, where $\leq$ is a binary relation symbol. We consider a class $\mathcal{K}$ of finite $\mathcal{L}$-structures $\mathcal{A} = (A, \leq)$ satisfying the following properties:

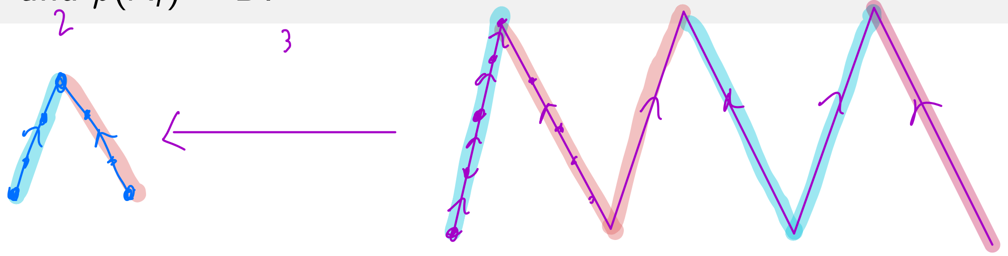
1. $\leq^{\mathcal{A}}$ is reflexive and antisymmetric.
2. $\mathcal{A}$ is connected.
3. Every element of $\mathcal{A}$ has degree one or two, and there are exactly two elements of $\mathcal{A}$ which have degree one.
   - If $x \in A$ has degree 1 and $\exists a \in A$ such that $a \leq^{A} x$, then say x is a **crown** of A.
   - If $x \in A$ has degree 1 and $\exists a \in A$ such that $x \leq^{A} a$, then say x is a **root** of A.
4. $\mathcal{A}$ has at least one root (in which case we say $\mathcal{A}$ is **rooted**), and we will designate one root a_0.

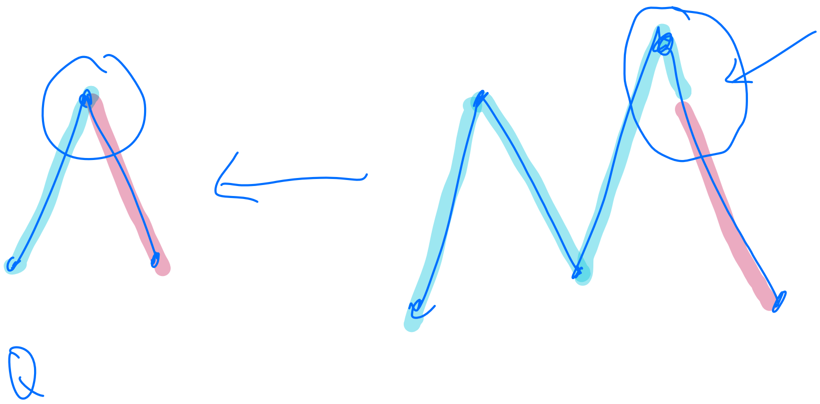
Describing the Approximating Class

Open Wrapping Epimorphisms

In an object $(A, \leq^{\mathcal{A}}) \in \mathcal{K}$, say $C \subseteq A$ is a directed chain if there exists a labeling on C, i.e. $C = \{c_0, ..., c_n\}$ for some $n \in$ with $c_i \leq^{\mathcal{A}} c_{i+1}$ for $0 \leq i \leq n-1$. Additionally, say $C \subseteq A$ is a maximal directed chain if C is a directed chain, and for any $a \in A \setminus C$, $C \cup \{a\}$ is not a directed chain. Finally, say an epimorphism $\rho : A \to B$ is an **open wrapping epimorphism** if every maximal directed chain $C_A \subseteq A$ maps to a maximal directed chain in B and $\rho^{-1}(B) = A_1 \cup A_2 \cup ... \cup A_k$ for some $k \in$ where each A_i has the same number of maximal directed chains as B and $\rho(A_i) = B$.

4





$c = f^{-1}(Q)$

$f(c) = Q$

confluent (but not open

Introduction to Knaster Continua

History

The Lattice

Abstract Characterization

Projective Fraïssé Theory

Projective Fraïssé Classes

Projective Fraïssé Limits,

Quotients of Projective Fraïssé Limits

Examples

Universal Knaster Continuum a Quotient of a Projective Fraïssé Limit

Approximating Class

Extreme Amenability

The Universal Minimal Flow

Quotient of the PF Limit

Con-Theorem, W. & Bartošová, 2021

Let $(\mathbb{K}, R^{\mathbb{K}})$ be the projective Fraïssé limit of $\mathcal{K}$, then:

1. $\mathbb{K}/R_S^{\mathbb{K}}$ is a continuum ✓
2. $\mathbb{K}/R_S^{\mathbb{K}}$ is arc-like, but not an arc ✓
3. Every proper subcontinuum of $\mathbb{K}/R_S^{\mathbb{K}}$ is an arc. ✓
4. $\mathbb{K}/R_S^{\mathbb{K}}$ satisfies the property of Kelley
5. $\mathbb{K}/R_S^{\mathbb{K}}$ has one end-point. ✓
6. For every K_α $\exists$ $\mathcal{K}_\alpha \subseteq \mathcal{K}$ s.t. $|\mathcal{K}_\alpha| = K_\alpha$

So $\mathbb{K}/R_S^{\mathbb{K}}$ maps openly onto every other Knaster continuum

Corollary

$\mathbb{K}/R_S^{\mathbb{K}}$ is a Knaster Continuum

$\to$ is K_p

Outline

Extreme Amenability of a Group

$G \curvearrowright M$ If M is a minimal G-flow (has no proper invariant subset)
then $M = \{*\}$

Definition

Let G be a topological group. We say G is extremely amenable if $M(G) = \{*\}$.

If the universal minimal G-flow is a single point, then that means every minimal G-flow must be a single point. Since every G-flow contains a minimal G-flow (Zorn), that means every G-flow X must contain a fixed point $x \in X$, i.e. $G \cdot x = x$.

Conversely, if every G-flow has a fixed point, then $M(G) = \{*\}$.

Automorphism Groups of Fraïssé Limits

Kechris, Pestov, Todorocevic, 2005

Let L be a signature with $L \subseteq \{<\}$ and let $\mathcal{K}$ be a Fraïssé order class in L. Let $\mathbb{F} = \mathrm{Flim}(\mathcal{K}$ be the Fraïssé limit of $\mathcal{K}$, so that **F** is an order structure. Then the following are equivalent:

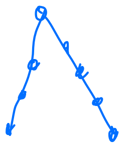
- $G = \mathrm{Aut}(\mathbf{F})$ is extremely amenable.
- $\mathcal{K}$ has the Ramsey property.

Structural Ramsey Property

Say that $\mathcal{K}$ satisfies the **Ramsey Property** if for every $A, B \in \mathcal{K}$ with $A \leq B$, and $k \geq 2$, there is $C \in \mathcal{K}$ such that:

$$C \to (B)^A_k$$

Homeo(K_ρ) is Extremely Amenable



Theorem, W. & Bartošová, 2021

The class $\mathcal{K}$ which approximates the universal Knaster continuum is Ramsey.

Corollary

The homeomorphism group of the universal Knaster continuum K_ρ, Homeo(K_ρ) is extremely amenable.

$K_\alpha \leftarrow$ α has infinitely many 2's
is Ramsey

Homeo$(K_2) \curvearrowright K_2$

$M(\text{Homeo}(K_3)) = \{0,1\}$

Procedure

Procedure For Model Theoretic Approach to Dynamics of a Compact Space:

1. Begin with some compact space Y. Aim: discover properties of Y and Homeo(Y) using finite approximations of Y.
2. Find a projective Fraïssé family which "apprxoimates" Y, call it $\mathcal{D}$.
3. Compute the projective Fraïssé limit of $\mathcal{D}$, call it $\mathbb{D}$.
4. Symmeterize the relations to get an equivalence relation. Then show that $\mathbb{D}/R_S^{\mathbb{D}} \simeq Y$.
5. The quotient map $\mathbb{D} \to \mathbb{D}/R_S^{\mathbb{D}}$ induces a continuous group embedding $\mathrm{Aut}(\mathbb{D}) \hookrightarrow \mathrm{Aug}(Y)$ with dense image.
6. We can study Homeo(Y) via the combinatorial properties of the finite structures in $\mathcal{D}$.

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Thank you!